

Developing and Using the ‘Knowledge Quartet’: A Framework for the Observation of Mathematics Teaching

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Abstract: This paper describes a framework for mathematics lesson observation and illustrates the ways that this framework is being used in practice, for mathematics teaching development. The research which led to the development of the framework drew on videotapes of mathematics lessons prepared and conducted by elementary pre-service students towards the end of their initial training. A grounded theory approach to data analysis led to the emergence of the framework - a ‘knowledge quartet’, with four broad dimensions, through which the mathematics-related knowledge of these teachers could be observed in practice. We term the four units: foundation, transformation, connection and contingency. This paper describes how each of these units is characterised, and analyses a fragment of one of the videotaped lessons, showing how each dimension of the quartet can be identified in the lesson.

Key words: Elementary teaching; Mathematics teaching; Teacher education; Teacher knowledge; Grounded theory

Introduction

The work of mathematics teachers is underpinned by a subtle blend of mathematics and pedagogy. There is more, of course, as those involved in Initial Teacher Education (ITT), students and tutors, are only too aware. Indeed, in his seminal work in the 1980s, Lee Shulman identified seven categories of teacher knowledge. Four of these are not subject specific (an example is ‘knowledge of learners’) but three of them focus explicitly on subject ‘content’ knowledge: subject matter knowledge, pedagogical content knowledge and curricular knowledge.

Subject matter knowledge (SMK) is knowledge of the content of the discipline per se (Shulman, 1986, p. 9), consisting both of substantive knowledge (the key facts, concepts, principles and explanatory frameworks in a discipline) and syntactic knowledge (the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community). Pedagogical content knowledge (PCK) is particularly difficult to define and characterise, conceptualising both the link and the distinction between knowing something for oneself and being able to enable others to know it. PCK consists of “the ways of representing the subject which makes it comprehensible to others...[it] also includes an understanding of what makes the learning of specific topics easy or difficult ...” (Shulman, 1986, p. 9). Curricular knowledge encompasses the scope and sequence of teaching programmes and the materials used in them.

How are these different kinds of ‘content’ knowledge acquired by teachers of mathematics? This question is addressed throughout this paper, but it is important to acknowledge that the acquisition of knowledge for teaching neither begins nor ends during initial training, although the training institution and the practicum placement schools are key environments for raising and refining the pedagogical awareness of beginning teachers. In particular, it is expected that trainee teachers will develop detailed ‘curricular knowledge’ in their placements in schools, so that they come to know the scope and sequence of the local or national curriculum. In England, primary school teachers’ practice in schools is also highly determined, at present, by a high-profile national initiative – the National Numeracy Strategy – which aims to raise standards of teaching and learning in mathematics (see DfEE, 1999, although a revision of this document is due in 2006/07). The means by which they develop a more detached, strategy-independent rationale for what they do and how they do it is more complex, but no less important.

Our research is located in a collaborative project involving researchers at three UK universities, under the acronym SKIMA (subject knowledge in mathematics). The work in the early stages of our collaboration focused on investigations into trainees’ subject matter knowledge and its relation to teaching. This has been reported elsewhere (e.g. Rowland, Martyn, Barber, & Heal, 2002; Goulding, Rowland, & Barber, 2002). The research reported in this paper was undertaken in collaboration with two SKIMA colleagues, Peter Huckstep and Anne Thwaites.

Context and Purpose of the Research

In the UK, the majority of trainee teachers are graduates (in a variety of disciplines) who follow a one-year, full-time course leading to a Postgraduate Certificate in Education (PGCE) in a university education department. All primary trainees are trained to be generalist teachers of the whole primary curriculum. Later in their careers, most take on responsibility for leadership in one curriculum area (such as mathematics) in their school, but, almost without exception, they remain generalists, teaching the whole curriculum to one class.

Over half of the PGCE year is spent teaching in schools under the guidance of a school-based mentor. Placement lesson observation is normally followed by a review meeting between a school-based teacher-mentor and the student-teacher. On occasion, a university-based tutor will participate in the observation and the review. Research shows that such meetings typically focus heavily on organisational features of the lesson, with very little attention to mathematical aspects of mathematics lessons (Brown, McNamara, Jones, & Hanley, 1999; Strong & Baron, 2004).

The purpose of the research reported in this paper was to develop an empirically-based conceptual framework for lesson reviews with a focus on the mathematics content of the lesson and the role of the trainee's mathematics SMK and PCK. Such a framework would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories, with a set of easily-remembered labels for those categories.

The focus of this particular research was therefore on the ways that teacher trainees' mathematics content knowledge - both SMK and PCK - can be observed to 'play out' in practical teaching during school-based placements. We wish to clarify at the outset that whilst we see certain kinds of knowledge to be desirable for elementary mathematics teaching, we are convinced of the futility of asserting what a beginning teacher, or a more experienced one for that matter, ought to know. Our interest is in what a teacher does know and believe, and how opportunities to enhance knowledge can be identified. We believe that the framework that arose from this research - we call it the 'knowledge quartet' - provides a means of reflecting on teaching and teacher knowledge, with a view to developing both.

Method

This study took place in the context of a one-year PGCE course, in which 149 trainees followed a route focusing either on the Early Years (pupil ages 3-8) or the Primary Years (ages 7-11). Six trainees from each of these groups were chosen for observation during their final school placement. Two mathematics lessons taught by each of these trainees were observed and videotaped, i.e. 24 lessons in total. The trainees were asked to provide a copy of their planning for the observed lesson. As soon as possible after the lesson (usually the same day) the observer/researcher wrote a brief (400-500 words) account of what happened in the lesson, so that a reader might immediately be able to contextualise subsequent discussion of any events within it. These 'descriptive synopses' were typically written from memory and field notes, with occasional reference to the videotape if necessary.

From that point, we took a grounded approach to the data for the purpose of generating theory (Glaser & Strauss, 1967). In particular, we identified in the videotaped lessons aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that they could be construed to be informed by a trainee's mathematics content knowledge - either SMK or PCK. We realised later that most of these related to choices made by the trainee, in their planning or more spontaneously. Each was provisionally assigned an 'invented' code (see below). These were grounded in particular moments or episodes in the tapes. This provisional set of codes was rationalised and reduced (e.g. eliminating duplicate codes and marginal events) by negotiation and agreement in the research team.

Our analyses took place after the assessment of the school-based placement had been taken place, and played no part in the assessment process. Our task was to look for issues relating to the trainees' mathematics SMK and PCK, and not to make summative assessments of teaching competence.

This inductive process generated the following set of 17 agreed codes.

adheres to textbook	identifying errors
anticipation of complexity	making connections
awareness of purpose	overt subject knowledge
choice of examples	recognition of conceptual appropriateness
choice of representation	responding to children's ideas
concentration on procedures	theoretical underpinning
decisions about sequencing	use of opportunities
demonstration	use of terminology
deviation from agenda	

Next, we revisited each lesson in turn and, after further intensive study of the tapes, elaborated each descriptive synopsis into an analytical account of the lesson. In these accounts, significant moments and episodes were identified and tagged with one of the codes, with appropriate justification and analysis concerning the role of the trainee's content knowledge in the identified passages, with links to relevant literature.

The identification of these fine categories was a stepping stone with regard to our intention to offer them to colleagues for their use, as a framework for reviewing mathematics teaching with trainees following observation. Yet we did not want a 17-point tick-list (like an annual car safety check), but preferred a less complex, more readily-understood scheme which would serve to frame an in-depth discussion between teacher and observer. The key to our solution was the recognition of an association between elements of subsets of the 17 codes, enabling us to group them (again by negotiation in the team) into four broad, superordinate categories, or 'units', which we have named (I) foundation (II) transformation (III) connection (IV) contingency. These four units are the dimensions or 'members' of what we now call the 'knowledge quartet'.

Each unit is composed of a small number of subcategories that we judged, after extended discussions, to be of the same or a similar nature. Specifically, the codes contributing to each of the four units is as follows:

FOUNDATION: adheres to textbook; awareness of purpose; concentration on procedures; identifying errors; overt subject knowledge ; theoretical underpinning; use of terminology.

TRANSFORMATION: choice of examples; choice of representation; demonstration.

CONNECTION: anticipation of complexity; decisions about sequencing; making connections; recognition of conceptual appropriateness.

CONTINGENCY: deviation from agenda; responding to children's ideas; use of opportunities.

Naturally, we are immersed in the process from which the codes emerged. We believe, however, that our names for the 17 individual codes are less important to other users of the 'quartet' than a broad sense of the general character and distinguishing features of each of four broad units. We shall attempt to enable readers' 'getting a sense' of these four units in a moment.

Conceptualisation and exemplification: the case of Chloë

The process by which we arrived at the four dimensions of the knowledge quartet was grounded and inductive, by constant comparison across 24 lessons. Our purpose in this paper is more illustrative than inductive. In the next section we shall give a succinct account of how we now conceptualise the character of each dimension of the quartet. Then we shall show how and where it can be applied by detailed reference to one of the 24 videotaped lessons. The trainee in question, Chloë, was teaching a Year 1/2 (pupil age 5-7) class a particular strategy for mental subtraction.

This process of application is, in the spirit of grounded theory, allied to verification and further elaboration of theory. That is to say, the close examination of Chloë's lesson could be seen as what Glaser and Strauss (1967) call 'theoretical sampling'. This, and other cases of theoretical sampling (in particular, scrutiny of lessons other than our core set of 24 lessons) suggests that the quartet is comprehensive as a tool for thinking about the ways that subject knowledge comes into play in the classroom. However, it will become apparent that many moments or episodes within a lesson can be understood in terms of two or more of the four units; for example, a contingent response to a pupil's suggestion might helpfully connect with ideas considered earlier. Furthermore, it could be argued that the application of subject knowledge in the classroom always rests on foundational knowledge.

Drawing on the extensive range of data from the 24 lessons, we offer here a brief conceptualisation of each unit of the knowledge quartet.

The Knowledge Quartet

The brief conceptualisation of the knowledge quartet which now follows draws on the extensive range of data referred to above. As we observed earlier, the practical application of the knowledge quartet framework depends more on teachers and teacher educators understanding the broad characteristics of each of the four dimensions than on their recall of the 17 constituent codes. Some aspects of the characterisation below will be clarified in our consideration of the lesson that we have singled out for attention later in this paper.

Foundation

The first member of the quartet is rooted in the foundation of the trainees' theoretical background and beliefs. It concerns trainees' knowledge, understanding and ready recourse to their learning in the academy, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use. This distinction relates directly to Aristotle's account of 'potential' and 'actual' knowledge. "A man is a scientist ... even when he is not engaged in theorising, provided that he is capable of theorising. In the case when he is, we say that he is a scientist in actuality." (Lawson-Tancred, 1998, p. 267). Both empirical and theoretical considerations have led us to the view that the other three units flow from a foundational underpinning.

A key feature of this category is its propositional form (Shulman, 1986). It is what teachers learn in their 'personal' education and in their 'training' (pre-service in this instance). We take the view that the possession of such knowledge has the potential to inform pedagogical choices and strategies in a fundamental way. By 'fundamental' we have in mind a rational, reasoned approach to decision-making that rests on something other than imitation or habit. The key components of this theoretical background are: knowledge and understanding of mathematics per se; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about why and how it is learnt.

In summary, this category that we call ‘foundation’ coincides to a significant degree with what Shulman (1987) calls ‘comprehension’, being the first stage of his six-point cycle of pedagogical reasoning.

Transformation

The remaining three categories, unlike the first, refer to ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching. They focus on knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself. At the heart of the second member of the quartet, and acknowledged in the particular way that we name it, is Shulman’s observation that the knowledge base for teaching is distinguished by “... the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful” (1987, p. 15, emphasis added). This characterisation has been echoed in the writing of Ball (1988), for example, who distinguishes between knowing some mathematics ‘for yourself’ and knowing in order to be able to help someone else learn it. As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). Our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils), and which follows from deliberation and judgement informed by foundation knowledge. This category, as well as the first, is informed by particular kinds of literature, such as the teachers’ handbooks of textbook series or in the articles and ‘resources’ pages of professional journals. Increasingly, in the UK, teachers look to the internet for bright ideas and even for ready-made lesson plans. The trainees’ choice and use of examples has emerged as a rich vein for reflection and critique. This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity.

Connection

The next category binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content – the learning, perhaps, of a concept or procedure. It concerns the coherence of the planning or teaching displayed across an episode, lesson or series of lessons. Mathematics is notable for its coherence as a body of knowledge and as a field of enquiry, and the cement that holds it together is reason. The pursuit of coherence and mathematical connections in mathematics pedagogy has been stimulated recently by the work of Askew, Brown, Rhodes, Wiliam and Johnson (1997); of six case study teachers found to be highly effective, all but one gave evidence of a ‘connectionist’ orientation. The association between teaching effectiveness and a set of articulated beliefs of this

kind lends a different perspective to the work of Ball (1990b) who also strenuously argued for the importance of connected knowledge for teaching.

In addition to the integrity of mathematical content in the mind of the teacher and his/her management of mathematical discourse in the classroom, our conception of coherence includes the sequencing of topics of instruction within and between lessons, including the ordering of tasks and exercises. To a significant extent, these reflect deliberations and choices entailing not only knowledge of structural connections within mathematics itself, but also awareness of the relative cognitive demands of different topics and tasks.

Contingency

Our final category concerns the teacher's response to classroom events that were not anticipated in the planning. In some cases it is difficult to see how they could have been planned for, although that is a matter for debate. In commonplace language this dimension of the quartet is about the ability to 'think on one's feet': it is about contingent action. The two constituent components of this category that arise from the data are the readiness to respond to children's ideas and a consequent preparedness, when appropriate, to deviate from an agenda set out when the lesson was prepared. Shulman (1987) proposes that most teaching begins from some form of 'text' - a textbook, a syllabus, ultimately a sequence of planned, intended actions to be carried out by the teacher and/or the students within a lesson or unit of some kind. Whilst the stimulus - the teacher's intended actions - can be planned, the students' responses can not.

Brown and Wragg (1993) group listening and responding together in a taxonomy of 'tactics' of effective questioning. They suggest that 'responding' moves are the lynch pins of a lesson, important in the sequencing and structuring of a lesson, and observe that such interventions are some of the most difficult tactics for newly qualified teachers to master. The quality of such responses is undoubtedly determined, at least in part, by the knowledge resource available to the teacher. For example, Bishop (2001, pp. 95-96) recounts a nice anecdote about a class of 9- and 10-year-olds who were asked to give a fraction between $\frac{1}{2}$ and $\frac{3}{4}$. One girl answered $\frac{2}{3}$, "because 2 is between the 1 and the 3, and on the bottom the 3 lies between the 2 and the 4". Bishop asks his readers how they might respond to the pupil. It is relevant here to suggest that such a response might be conditioned by whether they were aware of Farey sequences and mediants, or what heuristics were available to them to explore the generalisation inherent in the pupil's justification.

Chloë's Lesson

We now proceed to show how the knowledge quartet might be understood and applied in the observation and review of placement lessons. We shall do this by homing in on a short (14 minutes) portion of one of the 24 videotaped lessons. As we remarked earlier, Chloë, the trainee in question, was teaching a Year 1/2 (pupil age 5-7) class a particular strategy for mental subtraction. By focusing on this vignette we aim to maximise the possibility of the reader's achieving some familiarity with the scenario, with Chloë and a few of the children in her class. What is lost, of course, is any sense of how the quartet might inform reflection on the rest of her lesson. On the other hand, the passage we have selected would be, in itself, a valuable focus for some useful reflection in the post-lesson mentoring discussion.

Conforming to the English National Numeracy Strategy (NNS) guidance (DfEE, 1999), Chloë segments the lesson into three distinctive and readily-identifiable phases: the mental and oral starter; the main activity (an introduction by the teacher, followed by group work, with tasks differentiated by pupil ability); and the concluding plenary. The learning objective stated in Chloë's lesson plan is: "Children should be able to subtract 9, 11, 19 and 21 using the appropriate strategies". The lesson begins with a three-minute mental and oral starter, in which Chloë asks a number of questions such as 'How many must I add to 17 to make 20?', 'How many more than 7 is 10?', designed to test recall of complements of 10 and 20. There follows a 14-minute introduction to the main activity. Chloë reminds the class that in their previous lesson (which was taught by her mentor) they added 9, 11, 19 and 21 to various 1-digit and 2-digit whole numbers. Chloë demonstrates how to subtract these same numbers by subtracting 10 or 20 first, then adding or subtracting 1. She has a large, vertically-mounted 1-100 square, and models the procedure, moving a counter vertically and horizontally on the hundred square. She calls on children to assist her as 'teachers' in the demonstration. At the end of the demonstration, Chloë lists an example of each of the four subtractions on a whiteboard. The class then proceeds to 23 minutes' seatwork on differentiated worksheet exercises that Chloë has prepared. The 'more able' children subtract 19 and 21, the others subtract 9 and 11. Finally, she calls them together for a four-minute plenary, in which they consider $30 - 19$ and $43 - 21$ together.

Chloë's Lesson and the Knowledge Quartet

We now home in on the introduction to the main activity, to see how it might be perceived through the lens of 'the knowledge quartet'. This is typical of the way that the quartet can be used by observers – usually teacher-colleagues and teacher

educators - to identify for discussion various matters that arise from the lesson observation, and to structure reflection on the lesson.

Some possibilities for discussion with the trainee, and for subsequent reflection, are flagged below thus: **Discussion point.** We emphasise that the process of *selection* in the commentary which follows has been extreme. Nevertheless, it offers a realistic agenda for a typical, time-constrained post-lesson review meeting.

Foundation. Chloë's lesson plan refers to "appropriate strategies" for subtracting four near-multiples of 10, without recording what strategies she has in mind. It becomes clear that she will emphasise mental, sequential strategies, perhaps with some use of informal jottings (DfEE, 1999, p. 2/4). This is very much in keeping with the *National Numeracy Strategy*, which, following the Dutch RME (Realistic Mathematics Education) approach, emphasises mental calculation methods in the early grades. Sequential (or cumulative) strategies for two-digit addition and subtraction begin with one number (for subtraction, the minuend) and typically move up or down the sequence of integers in tens or ones. Split-tens methods, by contrast, partition both numbers into tens and units and operate on the two parts separately, before re-combining (e.g. Anghileri, 2000, pp. 62-65). The objective of the previous lesson (on adding near-tens) and the current one is taken directly from the NNS *Framework* (DfEE, 1999) teaching programme for Year 2:

Add/subtract 9 or 11: add/subtract 10 and adjust by 1. Begin to add/subtract 19 and 21: add/subtract 20 and adjust by 1. (p. 3/10)

These objectives are clarified by examples later in the *Framework*; such as

$58+21=79$ because it is the same as $58+20+1$; $70-11=59$ because it is the same as $70-10-1$

$24-9=15$ because it is the same as $24-10+1$; $35+19=54$ because it is the same as $35+20-1$ (p. 4/35)

The superficial similarity in these examples; captured in the NNS objective immediately above, is, we would suggest, deceptive. The differences between them can be articulated in terms of what Marton and Booth (1997) call 'dimensions of variation'. The dimensions in this case bring with them different kinds and levels of complexity, as follows.

Dimension 1: Addition or subtraction. In general terms, it might be thought that subtraction is the more demanding. Indeed, the first lesson of the two had dealt exclusively with addition, the second with subtraction.

Dimension 2: Near multiples of 10 or 20. Again, it seems reasonable to anticipate that adding/subtracting 20 is the more demanding. Indeed, Chloë has explicitly planned for the lower-attaining groups of pupils to work exclusively with 9 and 11.

Dimension 3: One more or one less than 10/20. Addition and subtraction of 11/21 entail a sequence of actions in the *same* direction i.e. aggregation or reduction; whereas 9/19 require a *change* of direction for the final unit i.e. compensation. Research confirms what might be expected, that the latter is less spontaneous and more demanding (e.g. Heirdsfield, 2001). Indeed, the compensation strategy for adding/subtracting 9 is, in lay terms, a ‘trick’.

Discussion point: what considerations determined Chloë’s choice of worksheet problems for the two ‘ability’ groups in the class?

Transformation. We pick out two factors for consideration relating to this dimension of the quartet (as usual, bearing in mind that they are underpinned by foundational knowledge). First, Chloë’s use of the 100 square as a model or representation of the sequence of two-digit positive integers. The 100 square is useful for representing ordinal aspects of the sequence, though with some discontinuities at the ‘ends’ of the rows, and particularly for representing the place-value aspects, although a 0-99 square arguably does this better (Pasternack, 2003). Chloë makes full use of the 100 square in her exposition, but is frequently dismissive of children’s use of the spatial language that it invites. For example, subtracting 9 from 70, she places the counter on 70:

- Chloë: Right, there’s 70. [...] From 70 I want to take away nine. What will I do? Rebecca?
- Rachel: Go up one.
- Chloë: No, don’t tell me what I’m gonna go up or move, tell me what I actually do.
- Rachel: Take away one.
- Chloë: Take away one to take away nine? No. Remember when we added nine we added ten first of all, so what do you think we might take away here? Sam.
- Simon: Ten.

This would seem to relate to the format of the NNS examples (above), which she follows in four ‘model’ solutions that she writes for reference on the board, e.g.

$$70 - 9 = ?, 70 - 10 + 1 = 61$$

Somewhat surprisingly, the children are forbidden to use 100 squares when they do the worksheet exercises. Chloë refuses a request from one child for a “number square”, saying, “I want you to work them out all by yourselves”. In fact, there is nothing in Chloë’s lesson plan to indicate that she had intended to use the 100 square in her demonstration.

Discussion point: What led Chloë to use the 100 square? What are its potential affordances - and constraints - for calculation relative to the symbolic recording in the NNS examples? Had she considered using an empty number line (e.g. Rousham, 2003) as an alternative way of representing the numbers and their difference, of clarifying when compensation is necessary, and why?

The other aspect of transformation that we select here concerns Chloë’s choice of examples. As we have observed, this has emerged as a rich vein for reflection and critique in every one of the 24 videotaped lessons. Space considerations restrict us to mentioning just one, in fact the first chosen to demonstrate subtraction, following the initial review of addition. Chloë chooses to subtract 19 from 70. We have already argued that subtracting 11 and 21 would be a more straightforward starting point. Moreover, 70 is on the extreme right boundary of the 1 to 100 square. After moving up two squares to 50, there is no ‘right one’ square: it is then necessary to move down and to the extreme left of the next row, so the neat ‘knights move’ is obscured, and the procedure unnecessarily complicated. We note that one of the NNS *Framework* examples (above) is $70 - 11$, and that all four of Chloë’s whiteboard template examples were of the form $70 - n$.

Discussion point: Was Chloë aware in-the-moment of the complication mentioned above, or did she anticipate it in her planning? Did the symbolic form in her written plan ($70 - 20 + 1$) perhaps obscure the consequences of her using the 100 square for this calculation?

Connection. Chloë makes explicit links with the previous lesson on adding near-multiples of 10, and reviews the relevant strategies at the start of this one. Her oral and mental starter, on complements to 10 and 20, essentially focuses on the concept of subtraction as comparison, whereas the strategy taught in the main activity is on change-separate, or ‘take away’, subtraction (Carpenter & Moser, 1983). Procedures associated with the two concepts tend to be based on strategies for counting on and counting back respectively (*ibid.*). Arguably Chloë could have encouraged some flexibility in the choice of such procedures, whereas she chose to prescribe exclusively forms of counting back in the main activity. The effect of her approach to differentiation for the different groups was to emphasise the similarity between 9

and 11 (needing an initial subtract-10) and between 19 and 21 (subtract-20), when the pairing of 11 and 21 (consistent reduction) and 9 and 19 (needing compensation) was an alternative form of connection.

Given her use of the 100 square to demonstrate the strategies, there was scope for some discussion of the links between vertical and horizontal spatial movements on the board and the tens-ones structure of the numbers under consideration. As we have remarked, she actively discouraged children's reference to the spatial analogue. It seemed that her attention was on conformity at the expense of flexibility and meaning-making.

Discussion point: discussion could usefully focus on the two subtraction concepts, how they relate to the first two phases of the lesson, and whether comparison strategies might offer useful alternatives to 'take-away with compensation', in the case of subtracting 9/19.

Contingency. A key component of our conceptualisation of this dimension of the quartet relates to how the teacher responds to unexpected or deviant ideas and suggestions from children in the lesson. There are no compelling distractions from Chloë's planned agenda for the lesson in this episode, although the child's question about using the number squares for the exercises might be a case in point. Various children's use of up/down language on the 100 square, to which we have already referred, might have been usefully explored rather than dismissed. A similar opportunity presented itself when, in the review of adding 9 at the beginning of the lesson, Chloë invites one of the pupils to demonstrate:

Chloë: Show the class how you add ten and take away one on a number square. What's the easy way to add ten on a number square?
Cameron.

Cameron: Go diagonally.

Chloë: Not diagonally. To add ten you just go...

Cameron: Down.

No further reference is made to Cameron's diagonal proposal, although his elegant use of vocabulary alone is surely worth a moment's pause. It is true that his initial suggestion is not, strictly, a correct answer to her "add ten" question. It does, however, offer a nice spatial way of thinking about adding 9 - and adding 11 too - and suggests that Chloë's mentor may have stressed it in the previous lesson. Indeed, the fact that adding 9 corresponds to a diagonal south-west move might usefully connect to the insight that subtracting 9 would necessitate a north-east move, and the consequent need to *add* one after subtracting 10. It would seem that

Chloë is too set on her own course to explore the possibilities offered by remarks such as Cameron's.

Discussion point: Did Chloë recall Cameron's suggestion? If so, how did she feel about it at the time, and how might she have responded differently?

It is important to add that the second question in this proposed discussion point is not intended as a thinly-veiled rebuke or correction: there are often very good reasons for teachers sticking to their chosen path. The purpose of the question is to raise awareness of the fact that an opportunity was presented, and that a different choice could have been made. We also reiterate that a single event or episode can frequently be considered from the perspective of two or more dimensions of the quartet, as demonstrated in our commentary.

Further Developments

In this paper, we have introduced 'the knowledge quartet' and shown its relevance and usefulness in our analysis of part of Chloë's lesson with a Year 1/2 class. We have a manageable framework within which to discuss actual, observed teaching sessions with trainees and their mentors. These groups of participants in initial teacher preparation, as well as our university-based colleagues, need to be acquainted with (and convinced of the value of) the quartet, and to be familiar with some details of its conceptualisation, as described in this paper. Within the last three years we have taken steps towards this familiarisation in the context of our own university's pre-service elementary and middle school teacher education programmes. The four dimensions of the knowledge quartet have been used as a framework for lesson observation and reflection. Appropriation of the framework by mentors has been facilitated through (a) training sessions at which the knowledge quartet is introduced and shown to have arisen from empirical research (b) a written version of the theoretical conceptualisation of the quartet, but framed as a set of questions to be addressed in lesson observation e.g. "Does the trainee select appropriate forms of representation e.g. the use of a number line when teaching subtraction by 'counting back' or a place value chart and arrow cards when teaching about values of digits" (c) group sessions at which videotapes of trainees' lessons are reviewed by mentors against the quartet in general and these 'prompt' questions in particular.

Initial indications are that this development has been well received by mentors, who appreciate the specific focus on mathematics content and pedagogy. They observe that it compares favourably with guidance on mathematics lesson observation from the NNS itself, which focuses on more generic issues such as "a crisp start, a well-planned middle and a rounded end. Time is used well. The teacher keeps up a

suitable pace and spends very little time on class organisation, administration and control.” (DfEE, 2000, p. 11).

It is all too easy for analysis of a lesson taught by a novice teacher to be (or be perceived to be) gratuitously critical, and we therefore emphasise that the quartet is intended as a tool to support teacher development, with a sharp and structured focus on the impact of their SMK and PCK on teaching. Indications of how this might work are explicit in our analysis of Chloë’s lesson. We have emphasised that our analysis has been selective: we raised for attention some issues, but there were others which, not least out of space considerations, we chose not to mention. The same would be likely to be true of the review meeting - in that case due to time constraints, but also to avoid overloading the trainee with action points. Such a meeting might well focus on a lesson fragment, and on only one or two dimensions of the knowledge quartet for similar reasons. At our university, mentor training on the knowledge quartet and its use has emphasised the need to be specific and selective in the use of feedback. Mentors took part in workshops in which several groups observed the same videotaped lesson with a focus on one of the four dimensions of the quartet. Useful discussions followed concerning the quantity and type of feedback that would be appropriate.

Any tendency to descend into deficit discourse is also tempered by consideration of the wider context of the student teacher’s experience in school. In the novice teacher we see the very beginnings of a process of reconciliation of pre-existing beliefs, new ‘theoretical’ knowledge, ‘practical’ advice received from various quarters, in the context of highly-pressured, high-stakes school-based placements. There is also good evidence (e.g. Hollingsworth, 1988; Brown, Mcnamara, Jones, & Hanley, 1999) that trainees’ concern for pupil learning is often eclipsed by their anxieties about timing, class management and pupil behaviour. In an attempt to recognise and address this, the knowledge quartet is being introduced to trainee teachers at our university, in order to direct attention to the subject content dimension of their classroom practice, and the ways that content knowledge might inform their planning, preparation and teaching. These sessions have also been well-received, with the trainees welcoming the quartet as a way of framing their thinking about their teaching. Many have expressed interest in participating in quartet-related research in their first teaching appointment (see below). By introducing mentors, mathematics co-ordinators and trainees to the knowledge quartet, we provide all relevant ‘stakeholders’ with a framework for the discussion of mathematics planning and teaching that will encourage a focus on the subject content as well as the management of lessons. In addition, we have recently been analysing lessons taught by secondary mathematics PGCE students, through the lens of the knowledge quartet. We could add that colleagues working in English and in science education

see potential in the quartet for their own lesson observations and review meetings: it would be interesting to see what the conceptualisations of the dimensions of the quartet might look like in these and other subject disciplines.

Our research on the application of the knowledge quartet as a tool to support teacher development is now being extended within a project working with beginning teachers during their initial training and the first three years of their teaching (see e.g. Turner, 2006). This project grows out of recognition that the mathematical knowledge and understandings of teachers, their beliefs about the nature of mathematics and the teaching and learning of mathematics, cannot be radically changed within the one-year PGCE. In this project the participants will be helped to become increasingly familiar with the quartet in order to use it as a shared language for discussion of, and reflection on, their mathematics teaching. To date, eleven trainees from one cohort of the Early Years and Primary PGCE course have worked with a researcher and discussed video-tapes of their lessons with specific reference to the four dimensions of the knowledge quartet. This is the beginning of a four year longitudinal study looking at how the quartet may be used with beginning teachers and their mentors to develop mathematics teaching. It is also expected that findings from this study will inform the development of the conceptualisation of the knowledge quartet and our understandings of how to facilitate its use by others.

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